Name of College - S.S. College, J-Bad
Deptt - Mathematics
Topic - Asymptotes
Class B.Sc. I (Hons) (Differential Cal.)
Time - 10.15 A.M to 11 A.M
11.00 A.M to 11.45 A.M

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Study Material -

Asymptote → It is a St line which cuts a given curve in two points at infinity but is not itself at infinity

or

An Asymptote is a tangent whose point of contact is (∞, ∞)

Working Rule for finding the asymptote :-
for Algebraic Curve.

(i) In the highest degree terms
put $x = 1$ and $y = m$.
then we get $\phi_n(m)$

(ii) $\phi_n(m) = 0$, calculate the values of m.

(iii) In the Next lower degree term.
put $x = 1$ $y = m$, we get
$\phi_{n-1}(m)$

(iv) To get c put the values of m in the formula

$$c = -\frac{\phi_{n-1}(m)}{\phi_n'(m)}$$

If this formula takes the form $\frac{0}{0}$ by substitution of the value of m

then use

$$\frac{c^2}{L2} f_n''(m) + c\, f_{n-1}'(m) + f_{n-2}(m) = 0$$

Asymptotes parallel to x-axis.

(1) Equate zero the co-efficient of the highest powers of x.

If the curve is of nth degree and the term of x^n be absent then equate the co-efficient of $x^{n-1} = 0$

the terms

If x^n and x^{n-1} be both absent then equate the co-efficient of $x^{n-2} = 0$ which gives two asymptotes parallel to x-axis.

Similarly Apply same process to find the asymptotes parallel to y-axis

If the equation of curve be of nth degree and if a_0 the co-efficient of x^n and a_n the co-efficient of y^n be not-zero then there will be no asymptotes parallel either x-axis or y-axis.

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Problem: Find the real asymptotes of the curve

$$x^3 + y^3 = 3axy$$

Put $x=1$, $y=m$ in third degree term and equating

$1+m^3=0$ $\qquad \phi_3(m) = 1+m^3$

$\Rightarrow (m+1)(m^2-m+1)=0$ $\qquad \phi_3'(m) = 3m^2$

$\Rightarrow m+1=0$ or $m^2-m+1=0$

$\Rightarrow m=-1$ $\qquad m=\frac{1\pm\sqrt{3}i}{2}$

$\phi_2(m) = -3am$

$\therefore c = -\frac{\phi_2(m)}{\phi_3'(m)} = -\frac{-3am}{3m^2} = \frac{a}{m}$

When $m=-1$ $\quad c=-a$

Other two values of m are imaginary therefore, two asymptotes corresponding to these values of m are imaginary.

Hence the real asymptote is

$$y = -x - a$$

$$\Rightarrow x+y+a=0$$

2. Find the asymptotes of

$$(x^2-y^2)y - 2y^2 + 5x - 7 = 0$$

Solution :→ Let $y = mx + c$ be a asymptote of the curve.

Let $y = mx + c$ in the equation of the curve.
We have

$$x^2(mx+c) - (mx+c)^3 - 2(mx+c)^2 + 5x - 7 = 0$$

$$mx^3 + x^2c - m^3x^3 - 3m^2x^2c - 3mxc^2 - c^3 - 2m^2x^2 - 4mxc - 2c^2 + 5x - 7 = 0$$

$$\Rightarrow x^3(m - m^3) + x^2(c - 3m^2c - 2m^2) + x(5 - 3mc^2 - 4mc) - c^3 - 2c^2 - 7 = 0$$

Equating the co-efficient of x^3

$$g_3(m) = m - m^3$$

$$g_2(m) = c - 3m^2c - 2m^2$$

$$g_1(m) = 5 - 3mc^2 - 4mc$$

$$g_0(m) = -c^3 - 2c^2 - 7$$

Now $g_3(m) = m - m^3 = 0$

$$\Rightarrow m(1 - m^2) = 0$$

$$\Rightarrow m = 0 \text{ or } m^2 - 1 = 0 \Rightarrow m = \pm 1$$

$$g_2(m) = c - 3m^2c - 2m^2 = 0$$

$$c(1 - 3m^2) = 2m^2$$

$$\therefore c = \frac{2m^2}{1 - 3m^2}$$

When $m = 0$, $c = 0$

When $m = 1$, $c = \frac{2}{-2} = -1$

When $m = -1$, $c = -1$.

Hence the Required Asymptotes

$y = mx + c$ $\quad m = 0, c = 0$

$\Rightarrow y = 0$

when $m = 1$ $c = -1$

$y = x - 1$

$\Rightarrow x - y - 1 = 0$

when $m = -1$ $c = -1$

$y = -x - 1$

$\Rightarrow x + y + 1 = 0$

$\therefore$ Required Asymptotes are

$y = 0$

$x - y - 1 = 0$

$x + y + 1 = 0$

Find the asymptotes of

$x^3 - 6x^2y + 11xy^2 - 6y^3 + x + y + 1 = 0$

Here 3rd degree term

$x^3 - 6x^2y + 11xy^2 - 6y^3$

Put $x = 1$, $y = m$

$g_3(m) = 1 - 6m + 11m^2 - 6m^3$

for values of m

$g_3(m) = 0$

$1 - 6m + 11m^2 - 6m^3 = 0$

$\Rightarrow 6m^3 - 11m^2 + 6m - 1 = 0$

$\Rightarrow 6m^3 - 6m^2 - 5m^2 + 5m + m - 1 = 0$

$\Rightarrow 6m^2(m-1) - 5m(m-1) + (m-1) = 0$

$\Rightarrow (m-1)(6m^2 - 5m + 1) = 0$

Either $m-1=0$ or $6m^2-5m+1=0$

$\Rightarrow m=1$ $\quad$ $m = \frac{5 \pm \sqrt{25-24}}{12}$

$= \frac{5 \pm 1}{12}$

$= \frac{1}{2}, \frac{1}{3}.$

Here $g_2(m) = 0.$

therefore $c = -\frac{g_2(m)}{g_3'(m)} = 0$

Now when $m=1$ $c=0$

Asymptote $y = x$ $\Rightarrow x-y=0$

When $m = 1/2$ $c=0$

$y = \frac{1}{2}x$ $\Rightarrow x-2y=0.$

When $m = 1/3$ $c=0$ $\Rightarrow$

Asymptote

$y = 1/3 x$

$\Rightarrow x-3y=0$

therefore required Asymptotes are

$x-y=0$

$x-2y=0$

$x-3y=0$