Motion of charged particle in uniform Electric field

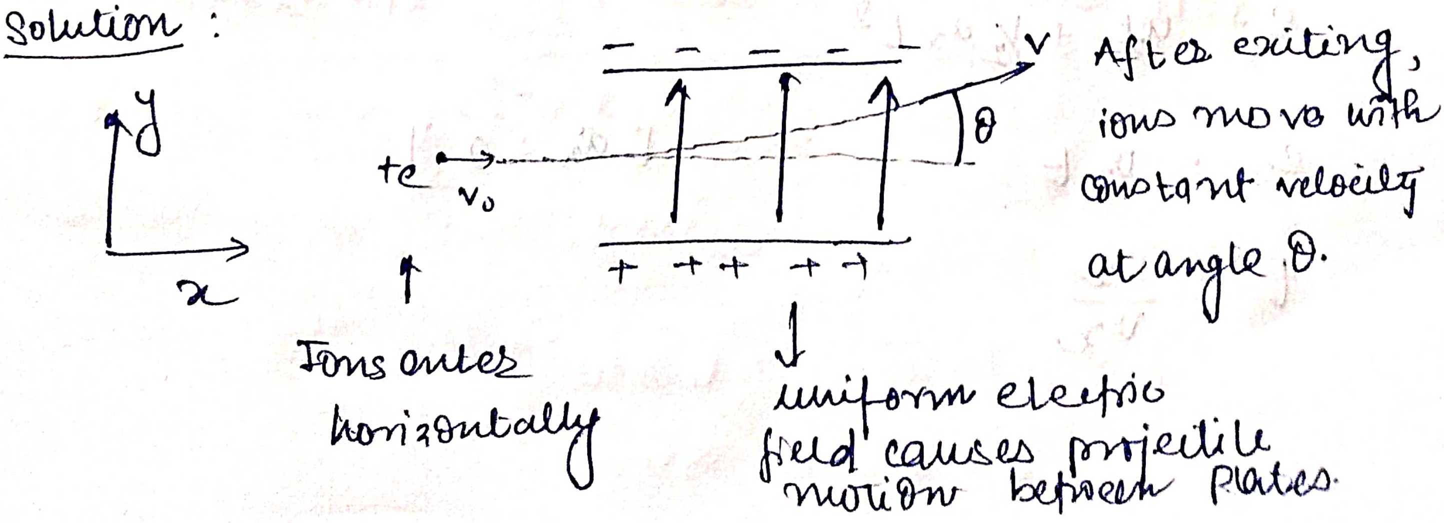
A charged particle in electric field $\vec{E}$, will exprience an electric force $\vec{F} = q\vec{E}$.

So, The acceleration of particle $\vec{a} = \frac{d\vec{v}}{dt}$

velocity of particle $\vec{v} = \frac{d\vec{r}}{dt}$

Problem: An ion milling machine uses a beam of gallium ions ($m = 70u$) to carve microstructures from a target. A region of uniform electric field between parallel sheets of charge is used for precise control of the beam direction. Singly ionized gallium atoms with an initially horizontal velocity of 1.8×10^4 m/s enter a 2.0 cm long region of uniform electric field which points vertically upward. The ions are redirected by the field and exit the field region at angle θ. If field is set to a value of $E = 90$ N/C what is the exit angle θ?

Solution:



On exiting point velocity vector makes an angle θ with the horizontal;

$$\Rightarrow \tan\theta = \frac{v_y}{v_x}$$

v_y is 'y' component of velocity

v_x is 'x' component of velocity

as electric field is applied in y direction

so acceleration will ~~have~~ be in y direction

no acceleration will be in x direction

v_x will be constant [velocity in x direction]

$v_y = a_y t$ [v_y velocity in y direction

a_y acceleration in y direction]

$$\Rightarrow t = v_y / a_y$$

Distance in x direction

$$S_x = u_x t + \tfrac{1}{2} a_x t^2$$

$$S_x = v_x t \qquad [a_x = 0]$$

$$t = \frac{S_x}{v_x}$$

$$\Rightarrow v_y = a_y \frac{S_x}{v_x}$$

$$a_y = \frac{F_y}{m} = \frac{eE}{m}$$

$$v_y = \frac{eE}{m} \frac{S_x}{v_x}$$

$$\tan\theta = \frac{v_y}{v_x} = \frac{\frac{eE}{m}\frac{S_x}{v_x}}{v_x} = \frac{eE}{m}\frac{S_x}{v_x^2}$$

$m = 70u$

$1u = 1.66 \times 10^{-27}$ kg

$m = 70 \times 1.66 \times 10^{-27}$ kg

$S_x = 2 \times 10^{-2}$ m [S_x = 2cm given]

$v_x = 1.8 \times 10^{4}$ m/s

$e = 1.6 \times 10^{-19}$ C

$$\therefore \tan\theta = \frac{1.6 \times 10^{-19} C \times 90 N/C \times (2 \times 10^{-2} m)}{70 \times 1.66 \times 10^{-27} kg \times (1.8 \times 10^{4})^2}$$

$\tan\theta = 7.6 \times 10^{-3}$

$\theta = \tan^{-1}(7.6 \times 10^{-3})$

$\simeq 44°$