Name of collage - S.S. Collage J. Bad
Depti - Mathematics
Topic - Plane (Analytical Geometry of)
class - B.Sc (Sub + Hons) 3rd
Time - 1:00 P.M to 1:45 Date 05-08-2020
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1 Study Material :- General Equation of plane

$$ax + by + cz + d = 0$$

(i) Here a, b, c are direction ratio of a st. line Normal to the plane.

(ii) $l = \dfrac{a}{\sqrt{a^2+b^2+c^2}}$ $m = \dfrac{b}{\sqrt{a^2+b^2+c^2}}$ $n = \dfrac{c}{\sqrt{a^2+b^2+c^2}}$

are direction cosine of a st. line Normal to the plane.

(iii) This plane passes through three points $\left(-\frac{d}{a}, 0, 0\right)$, $\left(0, -\frac{d}{b}, 0\right)$, $(0, 0, -d/c)$

(iv) Length of Intercept on x-axis $= -d/a$
Intercept on y-axis $= -d/b$
Intercept on z-axis $= -d/c$

Equation of plane in the Normal form

2. Normal form of the plane →

$$lx + my + nz = p$$

l, m, n are direction cosine of Normal
p = length of Normal from the origin on the plane. (Positive)

3. Incept Form → Equation of plane in the Intercept form →

$$\frac{x}{a} + y/b + z/c = 1$$

Where a, b, c are intercept cuts off on x-axis, y-axis and z-axis respectively

4. (i) Equation of Plane passing through three given points (x_1, y_1, z_1) (x_2, y_2, z_2) and (x_3, y_3, z_3) is given by

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

(ii) Four points (x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3) and (x_4, y_4, z_4) are coplanar if

$$\begin{vmatrix} x_4-x_1 & y_4-y_1 & z_4-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

5. Transformation of General Equation of plane

$$ax + by + cz + d = 0$$

in the Normal form

$$lx + my + nz = p$$

Solution ->

Since the equations

$$ax + by + cz = -d$$

and $$lx + my + nz = p$$

represent the same plane

$$\therefore \frac{a}{l} = \frac{b}{m} = \frac{c}{n} = \frac{-d}{p} = k$$

$\Rightarrow a = lk \qquad d = -pk$

$b = mk$

$c = nk$

Also $a^2+b^2+c^2 = (l^2+m^2+n^2)K^2$

Since $l^2+m^2+n^2 = 1$

$\therefore K^2 = a^2+b^2+c^2$

$\Rightarrow K = \pm\sqrt{a^2+b^2+c^2}$

$\therefore l = \frac{a}{K} = \pm\frac{a}{\sqrt{a^2+b^2+c^2}}$

Similarly $m = \pm\frac{b}{\sqrt{a^2+b^2+c^2}}$

$n = \pm\frac{c}{\sqrt{a^2+b^2+c^2}}$ $\quad p = -\frac{d}{\sqrt{a^2+b^2+c^2}}$

If d be +ve

$l = -\frac{a}{\sqrt{\Sigma a^2}}$ $\quad m = -\frac{b}{\sqrt{\Sigma a^2}}$ $\quad n = -\frac{c}{\sqrt{\Sigma a^2}}$

If d be -ve then

$l = \frac{a}{\sqrt{a^2}}$ $\quad m = \frac{b}{\sqrt{\Sigma a^2}}$ $\quad n = \frac{c}{\sqrt{\Sigma a^2}}$

Where $\Sigma a^2 = a^2+b^2+c^2$

Thus Normal form of the equation
$ax+by+cz+d=0$ is

$$-\frac{ax}{\sqrt{\Sigma a^2}} - \frac{by}{\sqrt{\Sigma a^2}} - \frac{cz}{\sqrt{\Sigma a^2}} = \frac{d}{\sqrt{\Sigma a^2}}$$

Angle between two planes →

Angle between two planes is equal to the angle between the normals drawn to the planes from any point in the space

for, if the equation of the planes be

$a_1 x + b_1 y + c_1 z = 0$

and $a_2 x + b_2 y + c_2 z = 0$

then the direction ratios are a_1, b_1, c_1

and a_2, b_2, c_2

∴ the actual direction cosines of the normals are

$$\pm \frac{a_1}{\sqrt{\Sigma a_1^2}}, \pm \frac{b_1}{\sqrt{\Sigma a_1^2}}, \pm \frac{c_1}{\sqrt{\Sigma a_1^2}}$$

and $$\pm \frac{a_2}{\sqrt{\Sigma a_2^2}}, \pm \frac{b_2}{\sqrt{\Sigma a_2^2}}, \pm \frac{c_2}{\sqrt{\Sigma a_2^2}}$$

$$\therefore \theta = \cos^{-1}\left[\frac{\pm a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{\Sigma a_1^2 \, \Sigma a_2^2}}\right]$$

+ sign gives the acute angle between given two plane.

these are mutually perpendicular if

$$a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$$

these are parallel if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

8 *. Angle between a plane and a st. line

If θ be the angle between a st line AB and a plane, then the angle between line AB and normal to the plane is $90-\theta$

Let $a_1 x + b_1 y + c_1 z + d_1 = 0$ be given plane

$\Rightarrow$ Direction ratios of the Normal to the plane is a_1, b_1, c_1

and let the direction ratio of line AB be a_2, b_2, c_2

then $\cos(90-\theta) = \dfrac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{\Sigma a_1^2 \, \Sigma a_2^2}} = \sin\theta$

* Let $lx + my + nz = p$ be the equation of the Given plane

then the equation of plane perpendicular to x-axis

$x = p$

The equation of plane perpendicular to x-axis

$y = p$

The equation of plane perpendicular to z-axis

$z = p$