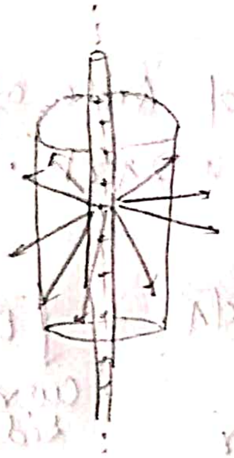
Cylindrical Gaussian Surface

Finding the electric field of an infinitely long, thin straight rod of charge, with a uniform linear charge density λ.



Now we will calculate the electric field using Gauss' Law. The only direction consistent with a cylindrically symmetric charge distribution is radial. Furthermore, the rotational symmetry around the rod dictates that the electric field has the same magnitude at any point on the surface of a cylinder concentric with the rod.

To apply Gauss' Law, we imagine a cylindrical surface with radius r and some length L.

No electric flux intercepts top or bottom. Electric field is radial intercepting curved area $A = 2\pi r L$ normally.

Inside our Gaussian surface, charge on rod is $Q_{inside} = \lambda L$

Here Gaussian surface an imaginary cylindrical surface of radius r and arbitrary length L.

The closed cylindrical surface consists of two circle (top and bottom) and a curve side. Since the electric field is radial, it is parallel to the top and bottom circular surfaces, so ~~net~~ no electric flux intercepted by these surfaces.
Net flux is contributed due to curved side of cylindrical surface which has area $A = 2\pi rL$.

Net flux $\oint \vec{E} \cdot d\vec{A} = \int_{\text{top circular}} \vec{E} \cdot d\vec{A} + \int_{\text{bottom circular}} \vec{E} \cdot d\vec{A} + \int_{\text{curved side}} \vec{E} \cdot d\vec{A}$

$\Rightarrow \quad \phi_E = \oint \vec{E} \cdot d\vec{A} = 0 + 0 + E \times 2\pi r L$

Gauss' Law

$\phi_E = \dfrac{Q_{inside}}{\epsilon_0}$

$\Rightarrow \quad E \times 2\pi r L = \dfrac{\lambda \times L}{\epsilon_0}$

$\Rightarrow \quad E = \dfrac{\lambda}{2\pi \epsilon_0 r}$.