Divergence

Del is denoted by ∇

$$\vec{\nabla} = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$$

When del operate

$\vec{\nabla} T$ called gradient here T is scalar function

$\vec{\nabla} \cdot \vec{v}$ called divergence here $\vec{v}$ is vector function

$\vec{\nabla} \times \vec{v}$ called curl here $\vec{v}$ is vector function.

Now $\vec{\nabla} \cdot \vec{v} = \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right) \cdot (v_x\hat{x} + v_y\hat{y} + v_z\hat{z})$

$$= \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

The divergence of a vector function v is itself a scalar.

Eg. Suppose the function $\vec{v}_a = x\hat{x} + y\hat{y} + z\hat{z}$

$\vec{v}_b = \hat{z}$

$\vec{v}_c = z\hat{z}$

Calculate divergence?

Solution $\vec{\nabla} \cdot \vec{v}_a = \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right) \cdot (x\hat{x} + y\hat{y} + z\hat{z})$

$$= \frac{\partial x}{\partial x} + \frac{\partial y}{\partial y} + \frac{\partial z}{\partial z} = 1 + 1 + 1 = 3$$

$$\vec{\nabla} \cdot \vec{v}_b = \frac{\partial 0}{\partial x} + \frac{\partial}{\partial y}0 + \frac{\partial}{\partial z}1 = 0$$

$$\vec{\nabla} \cdot \vec{v}_c = \left(\frac{\partial 0}{\partial x} + \frac{\partial}{\partial y}0 + \frac{\partial}{\partial z}z\right) = 1$$

Eq.2 $V_a = x^2\hat{x} + 3xz^2\hat{y} - 2xz\hat{z}$

Calculate divergence ?

Solution

$\vec{\nabla} \cdot \vec{V_a}$

$= \left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}\right) \cdot \left(x^2\hat{x} + 3xz^2\hat{y} - 2xz\hat{z}\right)$

$= \frac{\partial}{\partial x}x^2 + \frac{\partial}{\partial y}(3xz^2) - \frac{\partial}{\partial z}(2xz)$

$= 2x - 2x = 0$

Eq.3 $\vec{V_b} = xy\hat{x} + 2yz\hat{y} + 3zx\hat{z}$

Calculate divergence ?

Solution $\vec{\nabla} \cdot \vec{V_b}$

$= \left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}\right) \cdot (xy\hat{x} + 2yz\hat{y} + 3zx\hat{z})$

$= \left(\frac{\partial}{\partial x}xy + \frac{\partial}{\partial y}2yz + \frac{\partial}{\partial z}3zx\right)$

$= y + 2z + 3x = 3x + y + 2z$