Name of College - S.S. College J-Bad
Deptt - Mathematics
Topic - Problem based of Asymptotes Oblique Algebraic curve. (Cont-d)
Class - B.Sc I (Hons)
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Problem 1. Find the Asymptotes of the Curve.

$$x^4 - y^4 + 2ax^2y = b^2x^2.$$

Solution → Here By the ~~quetatio~~ question Equation of the curve is

$$x^4 - y^4 + 2ax^2y = b^2x^2.$$

This Equation can be written as

$$(x^2-y^2)(x^2+y^2) + 2ax^2y - b^2x^2 = 0$$

$$\Rightarrow (x-y)(x+y)(x^2+y^2) + 2ax^2y - b^2x^2 = 0$$

The Asymptotes parallel to $x-y=0$

$$x - y + \lim_{\substack{x,y\to\infty \\ y/x\to 1}} \frac{2ax^2y - b^2x^2}{(x+y)(x^2+y^2)} = 0$$

$$\Rightarrow x-y+\lim_{\substack{x,y\to\infty\\ y/x\to 1}} \frac{2a\, y/x - b^2\cdot\frac{1}{x}}{(1+y/x)(1+y^2/x^2)} = 0$$

$$\Rightarrow x-y+\frac{2a-b^2 0}{(1+1)(1+1)} = 0$$

$$\Rightarrow x-y+\frac{2a}{4} = 0$$

$$\Rightarrow x-y+\frac{1}{2}a = 0$$

The Asymptotes parallel to $x+y=0$

is $$x+y+\lim_{\substack{x,y\to\infty\\ y/x\to -1}} \frac{2a\cdot y/x - b/x}{(1-y/x)(1+y^2/x^2)} = 0$$

$$\Rightarrow x+y+\frac{2a(-1)}{(1+1)(1+1)} = 0$$

$$\Rightarrow x+y-\frac{2a}{4} = 0$$

$$\Rightarrow x+y-\frac{a}{2} = 0$$

Hence the required Asymptotes are

$$x+y+\frac{1}{2}a = 0$$

$$x+y-\frac{1}{2}a = 0$$

Find the Asymptotes of the Curve

$$(x^2-y^2)^2 - 2a^2(x^2+y^2) + xa^3 = a^4$$

Solution

Here equation of the curve is

$$(x^2-y^2)^2 - 2a^2(x^2+y^2) + xa^3 = a^4$$

it can be written as.

$$(x+y)^2(x-y)^2 = 2a^2(x^2+y^2) - xa^3 + a^4$$

Thus the Asymptotes Parallel to $x+y=0$ is

$$x+y = \pm\sqrt{\lim_{\substack{x,y\to\infty \\ y/x\to -1}} \frac{2a^2(x^2+y^2) - xa^3 + a^4}{(x-y)^2}}$$

$$= \pm\sqrt{\lim_{\substack{x,y\to\infty \\ y/x\to -1}} \frac{2a^2(1+y^2/x^2) - \frac{a^3}{x} + \frac{a^4}{x^2}}{(1-y/x)^2}}$$

$$= \pm\sqrt{\frac{2a^2(1+1) - 0 - 0}{2^2}}$$

$$= \pm\sqrt{\frac{2a^2 \times 2}{4}} = \pm a$$

∴ The Asymptotes parallel to $x+y=0$ is

$x+y-a=0$

$x+y+a=0$

Again the Asymptotes parallel to $x-y=0$ are given by

$$x-y=\pm\sqrt{\lim_{\substack{x,y\to\infty\\ y/x\to 1}}\frac{2a^2(x^2+y^2)-xa^3+a^4}{(x+y)^2}}$$

$$=\pm\sqrt{\lim_{\substack{x,y\to\infty\\ y/x\to 1}}\frac{2a^2(1+y^2/x^2)-\frac{a^3}{x}+\frac{a^4}{x^2}}{(1+y/x)^2}}$$

$$=\pm\sqrt{\frac{2a^2\times 2}{4}}$$

$$=\pm a$$

$\therefore$ Asymptotes parallel to $x-y=0$ are

$$x-y+a=0$$
$$x-y-a=0$$

thus Total No of Asymptotes are

$x+y-a=0$ $\quad x-y-a=0$

$x+y+a=0$ $\quad x-y+a=0$ $\quad$ Either

Which are parallel to $x-y=0$

or $x+y=0$