Name of College. S. S. College,
J. Bad.
Deptt — Mathematics

Topic → Plane (Analytical Geometry of 3-D)

Class - B.Sc I (Sub)

Time — 1.05 P.M to 1.45 P.M

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Study material → 1. General Equation of First degree in x, y, z is

$$ax + by + cz + d = 0$$

Where a, b, c, d are Constants and at least One of $a, b,$ and c is non-Zero.

It represent a plane.

a, b, c are direction ratio of the Normal from the origin on the Normal

this plane passing through plane these points are $\left(-\frac{d}{a}, 0, 0\right), \left(0, -\frac{d}{b}, 0\right), \left(0, 0, -\frac{d}{c}\right)$

Intercept on x-axis $= -\frac{d}{a}$

Intercept on y-axis $= -d/b$

Intercept on z-axis $= -d/c$

Normal Form of the equation of a plane →

$$lx + my + nz = p$$

is the Normal Form of the equation of plane

Where p = length of the normal from the origin on the plane

l, m, n are the direction cosine of the normal

* Equation of plane passing through three non-collinear points is

$$\begin{vmatrix} x & y & z & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{vmatrix} = 0$$

or

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

* Equation of plane which cuts of intercept on axes a, b, c respectively

Since plane cuts of intercept on x-axis $= a$
on y-axis $= b$
on z-axis $= c$

thus it passes through the three points $(a,0,0)$, $(0,b,0)$ and $(0,0,c)$ respectively

thus ~~its~~ here $x_1 = a$ $y_1 = 0$, $z_1 = 0$

$x_2 = 0$ $y_2 = b$ $z_2 = 0$

$x_3 = 0$ $y_3 = 0$ $z_3 = 0$

this equation of the plane

using formula

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-a & y & z \\ -a & b & 0 \\ -a & 0 & c \end{vmatrix} = 0$$

$$\Rightarrow (x-a)\begin{vmatrix} b & 0 \\ 0 & c \end{vmatrix} - y\begin{vmatrix} -a & 0 \\ -a & c \end{vmatrix} + z\begin{vmatrix} -a & b \\ -a & 0 \end{vmatrix} = 0$$

$$\Rightarrow (x-a)bc - y(-ac) + z(+ab) = 0$$

$$\Rightarrow bcx - abc + acy + zab = 0$$

$$\Rightarrow bc\cdot x + acy + zab = abc$$

$$\Rightarrow abc\left(\frac{x}{a} + y/b + \frac{z}{c}\right) = abc$$

$$\Rightarrow \frac{x}{a} + y/b + z/c = 1$$

Condition under which four points (x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3) and (x_4, y_4, z_4) are coplanar —

$$\begin{vmatrix} x_4-x_1 & y_4-y_1 & z_4-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

in the Normal form
$$lx + my + nz = p$$

Since the two equations

$$ax + by + cz = -d \quad \text{——— (i)}$$
$$lx + my + nz = p \quad \text{——— (ii)}$$

represent the same plane

$$\Rightarrow \frac{l}{a} = \frac{m}{b} = \frac{n}{c} = \frac{p}{-d} = k$$

$$l = ak, \quad m = bk, \quad n = ck, \quad p = -dk$$

but $l^2 + m^2 + n^2 = 1$

$$(a^2 + b^2 + c^2) k^2 = 1$$

$$\therefore k^2 = \frac{1}{a^2 + b^2 + c^2}$$

$$\Rightarrow k = \pm \frac{1}{\sqrt{a^2 + b^2 + c^2}}$$

$$\therefore l = \pm \frac{a}{\sqrt{a^2+b^2+c^2}} \qquad m = \pm \frac{b}{\sqrt{a^2+b^2+c^2}} \qquad n = \pm \frac{c}{\sqrt{a^2+b^2+c^2}}$$

$$p = \pm \left(-\frac{d}{\sqrt{a^2+b^2+c^2}} \right)$$

Now putting the values of l, m, n in (ii)

Ex:→ Find the intercepts of the plane $2x - 4y + 5z = 20$

$$\Rightarrow \frac{2x}{20} - \frac{4y}{20} + \frac{5}{20}z = 1$$

$$\Rightarrow \frac{x}{10} + \frac{y}{-5} + \frac{z}{4} = 1$$

thus Required intercepts on the axes

10, −5, 4

Problem :→ A plane meets the co-ordinate axes in A, B, C such that the centroid of the ΔABC is the point (f, g, h). Find the equation of the plane.

Solⁿ:
Let the co-ordinates of the points A, B, C be respectively
$(a, 0, 0), (0, b, 0), (0, 0, c)$

therefore, the intercepts of the plane with the co-ordinate axes are a, b, c

thus the equation of plane is

$$\frac{x}{a} + y/b + z/c = 1 \qquad \text{———} \quad (1)$$

Since (f, g, h) is centroid

$$\Rightarrow f = \frac{a}{3}, \quad g = b/3 \quad h = c/3$$

$$\Rightarrow a = 3f \qquad b = 3g \qquad c = 3h.$$

From (1)

$$\frac{x}{3f} + \frac{y}{3g} + \frac{z}{3h} = 1$$

$$\Rightarrow \frac{x}{f} + y/g + \frac{z}{h} = 3$$