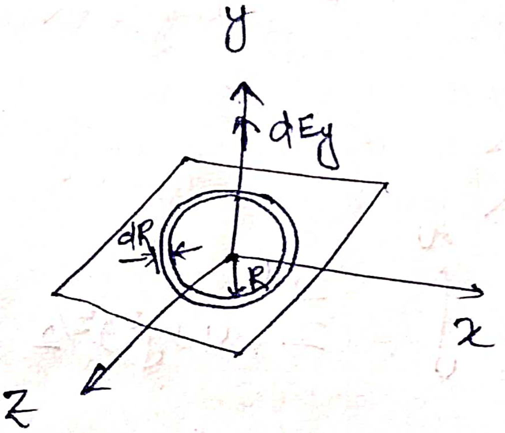
Problem : Positive charge is uniformly distributed over an infinite flat horizontal sheet, such as a very large sheet of paper. Suppose that the amount of charge per unit area on this sheet is a. Find the electric field in the space above and below the sheet.

Solution : ~~The~~



The sheet can be regarded as made up of many concentric rings. The above figure shows one of these rings, with radius R and width dR; this ring has an area dA = length × width

$$= 2\pi R \times dR = (2\pi R)dR$$

So, the small amount of charge possessed by the ring $dq = a\,dA$

$$= a\,2\pi R\,dR$$

$$dE_y = \frac{1}{4\pi\epsilon_0}\frac{dq}{(R^2+y^2)^{3/2}}$$ [Electric field produced by ring]

$$= \frac{1}{4\pi\epsilon_0}\frac{a\,2\pi R\,dR}{(R^2+y^2)^{3/2}}$$

The net electric field of the entire infinite sheet is then obtained by integrating over R, from R=0 to R=∞

$$E_y = \frac{ay}{2\varepsilon_0}\int_0^{\infty}\frac{R\,dR}{(R^2+y^2)^{3/2}}$$

$$u = R^2$$

$$du = 2R\,dR$$

$$E_y = \frac{ay}{2\varepsilon_0}\int_0^{\infty}\frac{du}{2(u+y^2)^{3/2}}$$

$$= \frac{ay}{2\varepsilon_0}\left[-\frac{1}{(u+y^2)^{1/2}}\right]_0^{\infty}$$

$$= \frac{ay}{2\varepsilon_0}\times\frac{1}{y} = \frac{a}{2\varepsilon_0}$$

$$\text{or} \quad E = \frac{a}{2\varepsilon_0}$$

This electric field is proportional to the charge density and it is constant, that is, it is independent of the distance from the sheet. Although the result is strictly valid only for the case of an infinitely large sheet, it is also a good approximation for a sheet of finite size, provided we stay near the sheet and we stay away from vicinity of the edge.