Name of College - S.S. College, J. Bad
Deptt - Mathematics
Topic - Asymptotes
Class - B.Sc I (HONS)

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Time - 10.15 A.M To 11.00 A.M        Date 17-08-2020
11.00 A.M To 11.45 A.M

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Problem - When the equation of curve be put in the form

$$(ax+by+c)P_{n-1} + Q_{n-1} = 0$$

Find the asymptotes of the curve.

$$(x+y)(x^4+y^4) = a(x^4+a^4)$$

Solution :→ The equation of the curve is of fifth degree and this can be written as

$$(x+y)P_4 + Q_4 = 0$$

This asymptote will be parallel to $x+y$

Thus the equation of asymptote will be.

$$x+y = \lim_{\substack{x\to\infty \\ y\to\infty}} \frac{a(x^4+a^4)}{x^4+y^4}$$ in the direc of $x+y=0$

Let $x = 1/t$, $y = -x = -1/t$

then $x \to \infty$, $t \to 0$
$y \to \infty$, $t \to 0$

$$x+y = a \lim_{t\to 0} \frac{\left(\frac{1}{t^4} + a^4\right)}{\frac{1}{t^4} + \frac{1}{t^4}} = a \lim_{t\to 0} \frac{1+a^4t^4}{2}$$

$$= \frac{a}{2}$$

$\therefore x+y = a/2.$

If the equation of curve be of the form

$(ax+by+c)P_{n-1} + S_{n-2} = 0$

Find the asymptotes of the curve

$(x+y)^2(x+2y+2) = x+9y+2$

When we put $x+2y+2=0$ in the given equation, then the equation of the curve becomes of the first degree

$x+9y+2=0$

that the term containing highest power 3 and the next highest power 2 becomes zero therefore,

$x+2y+2=0$ is the equation of an asymptote.

Also the given curve is of the form

$(x+y)^2 P_{n-1} + F_{n-1} = 0$

$$(x+y)^2 = \frac{x+9y+2}{x+2y+2}$$

the equations of the other two asymptotes will be

$$x+y = \pm \lim_{\substack{x+y=c \\ x\to\infty\, y\to\infty}} \sqrt{\frac{x+9y+2}{x+2y+2}}$$

Put $x = 1/t$ $y = -1/t$ so that $t \to 0$.

$$x+y = \pm \lim_{t \to 0} \sqrt{\frac{\frac{1}{t} + \frac{9}{t} + 2}{1/t - 2/t + 2}}$$

$$= \pm \lim_{t \to 0} \sqrt{\frac{1-9+2t}{1-2+2t}}$$

$$= \pm \lim_{t \to 0} \sqrt{-8/-1}$$

$$= \pm 2\sqrt{2}$$

$\therefore$

$$x+y-2\sqrt{2} = 0$$

$$x+y+2\sqrt{2} = 0$$

thus Required asymptotes are

$$x+2y+2 = 0$$

$$x+y-2\sqrt{2} = 0$$

$$x+y+2\sqrt{2} = 0$$

If the Equation of the curve is of the form

$$(ax+by)^2 P_{n-2} + (ax+b) Q_{n-2} + F_{n-2} = 0$$

Ex: $\to$ Find the asymptotes of the curve

$$(y-x)^2(y+x) - 6y(x-x) + 4x = 0$$

$$\Rightarrow (y-x)^2 - \frac{6y(y-x)}{y+x} + \frac{4x}{y+x} = 0$$

When $x \to \infty$, $y \to \infty$

then $x = 1/t$ in the direction of $y-x=0$

thus $y = 1/t$

$\therefore (y-x)^2 - 6(y-x)\lim_{t\to 0}\frac{1/t}{2/t} + 4\lim_{t\to 0}\frac{1/t}{2/t} = 0$

$\Rightarrow (y-x)^2 - 6(y-x)\times\frac{1}{2} + \frac{4}{2} = 0$

$\Rightarrow (y-x)^2 - 3(y-x) + 2 = 0$

$\Rightarrow (y-x)^2 - 3y + 3x + 2 = 0$

$\Rightarrow \cancel{y^2 - 2xy + x^2 - 3y + 3x + 2}$

$(y-x-1)(y-x-2) = 0$

$\Rightarrow y-x-1=0$

$y-x-2=0$

Again if we put $y+x=0$ in the given equation then the equation of the curve becomes one degree less. Therefore the other asymptotes will be parallel to $y+x=0$

From the equation of the curve

$$y+x - \frac{6y}{y-x} + \frac{4x}{(y-x)^2} = 0$$

the required asymptote will be

$y+x = \lim \frac{6y}{y-x} + \lim \frac{4x}{(y-x)^2}$ when $x\to\infty, y\to\infty$ in the direction of $y+x=0$

put $x=1/t$, $y=-1/t$

$\therefore$ $y+x = 6 \lim_{t\to 0} \dfrac{-1/t}{-1/t - 1/t} - 4 \lim_{t\to 0} \dfrac{1/t}{\left(-\frac{1}{t}-\frac{1}{t}\right)^2}$

$= 6 \times \frac{1}{2} - 4 \times 0$

$= 3$

$\therefore$ $y+x-3=0$

Therefore required Asymptotes are

$y-x-1=0$

$y-x-2=0$

and $y+x-3=0$.