Eq. A spherical region of radius R has a total charge Q, distributed uniformly throughout the volume of this region.

(a) what is the electric field at points inside the sphere?
(b) what is the electric field at points outside the sphere?

Solution:

(a) Since there is charge throughout the volume of the sphere, the electric field lines, charges distribution start within the volume of the spherical region, at varying distances from the center. Since the charge distribution is spherically symmetric, the electric field must be radial and constant.

To find the magnitude of electric field E at a radius $r<R$ inside the charge distribution, consider the spherical Gaussian surface of radius r.

Gauss' Law

$$\phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{inside}}{\epsilon_0}$$

$$\Rightarrow \int E\, dA \cos 0^\circ = Q_{inside}/\epsilon_0$$

$$\Rightarrow \int E\, dA = Q_{inside}/\epsilon_0$$

$$\Rightarrow E \times 4\pi r^2 = Q_{inside}/\epsilon_0 \quad -(1)$$

as area of sphere is $4\pi r^2$

To express Q_{inside} in term of the given quantities, we use the fact that the charge is uniformly distributed throughout the specified volume.

Thus for $r < R$ the charge ~~loss~~ is

$$Q_{inside} = \rho \times V_{inside} \quad -② $$

where the volume charge density ρ (charge per unit volume) is determined by the total charge Q and total volume V of the entire charged sphere:

$$\rho = \frac{Q}{V} = \frac{Q}{\frac{4\pi}{3}R^3} \quad -③$$

the quantity V_{inside} is the volume inside our Gaussian surface of radius r:

$$V_{inside} = \frac{4\pi}{3}r^3 \quad -④$$

As above mentioned

$$\therefore \quad Q_{inside} = \rho V_{inside}$$

$$= \frac{Q}{\frac{4\pi}{3}R^3} \times \frac{4\pi}{3}r^3 = Q\frac{r^3}{R^3} \quad -⑤$$

and from eqn ① we get

$$\Rightarrow E\,4\pi r^2 = \frac{Q_{inside}}{\epsilon_0}$$

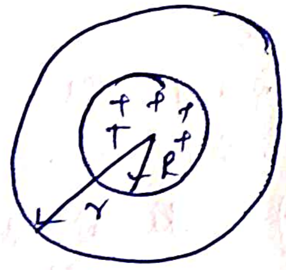
$$\Rightarrow E4\pi r^2 = \frac{Q r^3}{\epsilon_0 R^3}$$

$$\Rightarrow \boxed{E = \frac{Q}{4\pi\epsilon_0}\frac{r}{R^3} \quad (r < R)}$$

(b) To find the electric field outside the sphere, we again take a spherical Gaussian surface of radius r, but this time r is greater than R (radius of sphere)

According to Gauss' Law

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{inside}}{\epsilon_0}$$



From eqn (1)

$$E \times 4\pi r^2 = \frac{Q_{inside}}{\epsilon_0}$$

From figure it can be seen that charge inside $Q_{inside} = Q$ (Total charge of sphere)

$$\Rightarrow E = \frac{Q}{4\pi r^2 \epsilon_0} \qquad (r \geq R)$$

Graphical representation of variation of electric field with ~~radius~~ radial distance r.