Name of college — S. S. college, J. Bad page-1
Dept — Mathematics
Topic — Algebra of Matrices
class — B.Sc I (Hons+Sub) Time — 1.00 P.M to 1.45 P.M.
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Study Material

Equality of two Matrices —

Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be two matrices having the same order $m \times n$ are equal if $a_{ij} = b_{ij}$ for each $i = 1, 2, 3 \ldots$ and $j = 1, 2, 3 \ldots$

In other words, two matrices are said to be equal if they have the same order and their corresponding entries are equal.

Operations on Matrices

The Transpose of an $m \times n$ Matrix $A = [a_{ij}]$ is defined as the $n \times m$ Matrix $B = [b_{ij}]$ with

$$b_{ij} = a_{ji}$$

$$1 \leq i \leq m$$

$$1 \leq j \leq n$$

The Transpose of A is usually denoted by A^T or A'

Ex:→ $A = \begin{bmatrix} 1 & 4 & 5 \\ 0 & 1 & 2 \end{bmatrix}_{2\times3}$ $A^T = \begin{bmatrix} 1 & 0 \\ 4 & 1 \\ 5 & 2 \end{bmatrix}_{3\times2}$

Thus the Transpose of a Row vector is a column vector and vice-versa

For any matrix A,

$$(A^T)^T = A$$

2- <u>Addition of matrices</u> :→

Let $A = [a_{ij}]_{m\times n}$ and $B = [b_{ij}]_{p\times q}$

Then $A+B$ is calculated only if

$m = p$

$n = q$

i.e. order of both matrices should be equal

Then $A+B = [c_{ij}]_{m\times n}$ where $c_{ij} = a_{ij}+b_{ij}$

3- Multiplying a scalar to a matrix →

Let $A = [a_{ij}]$ be an $m\times n$ matrix

Then for every element $k \in R$

we define $kA = [ka_{ij}]$

For Example — if $A = \begin{bmatrix} 1 & 4 & 5 \\ 0 & 1 & 2 \end{bmatrix}_{2\times 3}$

$$\Rightarrow 5A = \begin{bmatrix} 5 & 20 & 25 \\ 0 & 5 & 10 \end{bmatrix}_{2\times 3}$$

Theorem → Let A, B, C be matrices of order $m\times n$ and let $k, l \in R$

Then (i) $A+B = B+A$

(ii) $(A+B)+C = A+(B+C)$

(iii) $k(lA) = kl(A)$

(iv) $(k+l)A = kA + lA$.

Multiplication of matrices

Definition – Matrix Multiplication / product.

Let $A = [a_{ij}]$ be $m \times n$ matrix and $B = [b_{ij}]$ be $n \times r$ matrix.

Then product AB is a matrix $C = [c_{ij}]_{m \times r}$

with $c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj} = a_{i1} b_{1j} + a_{i2} b_{2j} + \cdots + a_{in} b_{nj}$

Remarks – Product of AB is defined if and only the No of column of A = the No of Rows of B

Ex :→ if $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 1 \end{bmatrix}_{2 \times 3}$ and $B = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 3 \\ 0 & 0 & 4 \end{bmatrix}_{3 \times 3}$

Then $AB = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \end{bmatrix}_{2 \times 3}$

$c_{11} = 1 \times 1 + 2 \cdot 0 + 3 \times 1$
$= 1 + 0 + 3 = 4$

$c_{12} = 1 \cdot 2 + 2 \cdot 0 + 3 \cdot 0 = 2$

$c_{13} = 1 \cdot 1 + 2 \cdot 3 + 3 \cdot 4 = 19$

$c_{21} = 2 \cdot 1 + 4 \cdot 0 + 1 \cdot 1 = 3$ $c_{22} = 2 \cdot 2 + 4 \cdot 0 + 1 \cdot 0 = 4$

$c_{23} = 2 \cdot 1 + 4 \cdot 3 + 1 \cdot 4 = 18$

$= \begin{bmatrix} 4 & 2 & 19 \\ 3 & 4 & 18 \end{bmatrix}_{2 \times 3}$

NOTE → In this example, we see AB is defined, BA is not defined.
However, for square matrices A and B of the same order, both the product AB and BA are defined.

Two square matrices A and B are said to commute if $AB = BA$.

Remarks :→ If A is a square matrix of order n then $AI_n = I_nA$. Also for any $d \in R$ the matrix dI_n commutes with every square matrix of order n. The matrices dI_n for any $d \in R$ are called scalar matrices.

2. In general, the matrix product is not commutative.
For example, consider the following two matrices $A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

$$AB = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$$

Here $AB \neq BA$

$$BA = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$