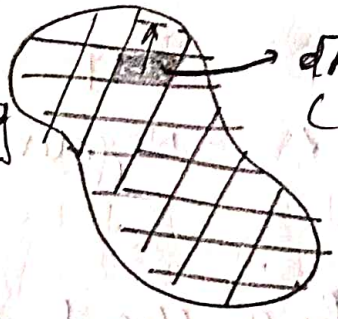
When electric field may have different magnitudes and directions at different points on the surface. Surface of arbitrary curved shape is choosen.

Here we considered surface as consisting of many small flat pieces.



($\vec{dA}$ vector $\vec{dA}$ has magnitude dA and direction perpendicular to surface)

For small piece of area dA, $d\phi_E$ is flux through the surface dA.

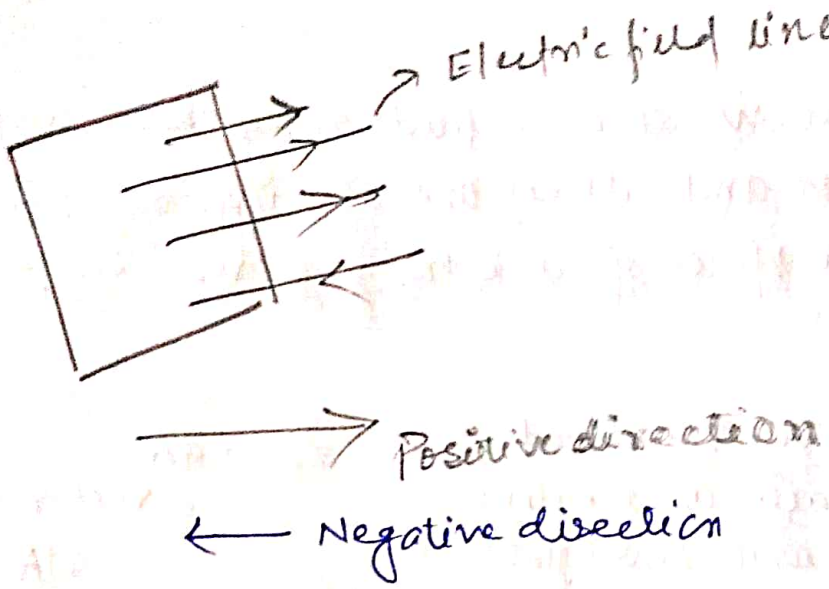
$$\therefore d\phi_E = \vec{E} \cdot d\vec{A}$$

Net electric flux over all surface calculated by summing or integrating these small amount of electric flux over the curve surface.

$$\therefore \phi_E = \int d\phi_E = \int \vec{E} \cdot d\vec{A}$$

Electric flux is proportional to the numbers of field lines crossing the surface.

For a surface, field lines going through the surface from one side make a positive contribution to the flux while as field lines going through from the other side make a negative contribution.



Electric flux through a closed surface is

$$\phi_E = \oint \vec{E} \cdot d\vec{A}$$

where circle on the integral sign indicates a closed surface