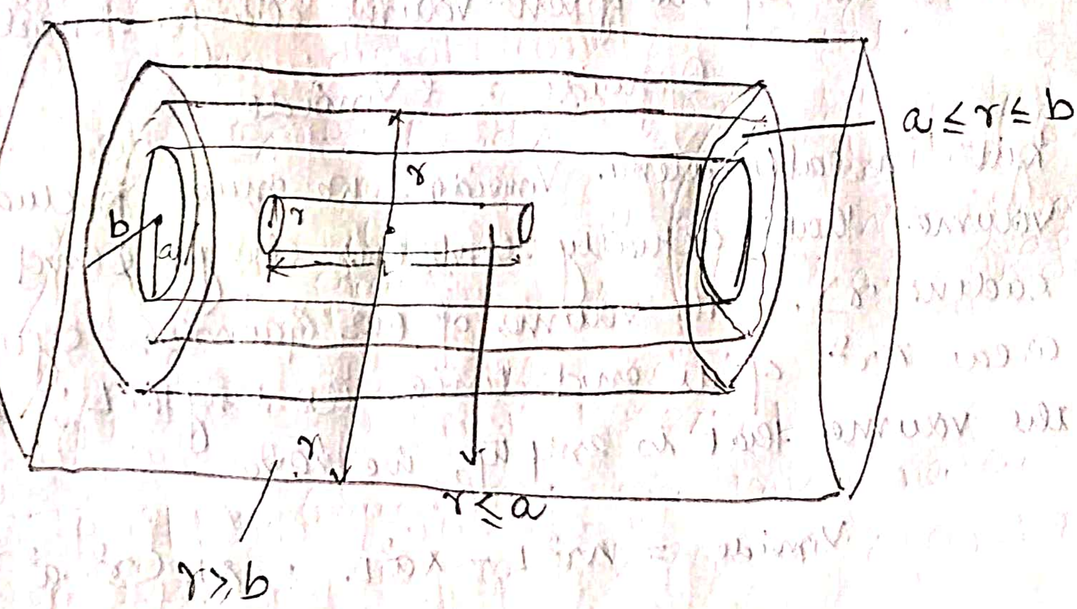
Ex. A thick insulating cylindrical shell with inner radius a and outer radius b has charge distributed throughout its volume, with a uniform volume charge density ρ.
Find the electric field in the three regions
(a) $r < a$ (b) $a \le r \le b$ (c) $r \ge b$

Solution:



Let Gaussian surface of radius 'r' and length 'L'.
By Gauss Law

$$\phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{inside}}{\epsilon_0}$$

Electric ~~e~~field is radial to the curved surface therefore Electric field is and area vector are ~~radially~~ in same direction.

$$E \cdot 2\pi r L = \frac{Q_{inside}}{\epsilon_0}$$

(a) For $r \le a$, consider the cylindrical Gaussian surface shown in ~~gree~~ figure.

Since it is contained completely inside the cylindrical hole within the shell, it encloses no charge. Thus $Q_{inside} = 0$

$$\therefore E = 0 \qquad (r \leq a)$$

(b) For $a \leq r \leq b$, consider a cylindrical Gaussian surface with its curved part inside the shell, as shown in figure. In terms of the given volume charge density ρ,

$$Q_{inside} = \rho V_{inside}$$

but in calculating V_{inside}, we must include only that volume that actually contains charge (and is inside the radius r). The volume of our Gaussian cylinder is the area πr^2 of its end times its length L; if we subtract the volume that is empty, we have

$$V_{inside} = \pi r^2 L - \pi a^2 L = \pi L (r^2 - a^2)$$

so, Gauss' Law becomes

$$E \, 2\pi r L = \frac{\rho V_{inside}}{\epsilon_0} = \frac{\rho \pi L (r^2 - a^2)}{\epsilon_0}$$

$$\Rightarrow \quad E = \frac{\rho \pi L}{\epsilon_0 \times 2\pi r L} (r^2 - a^2)$$

$$E = \frac{\rho}{2\epsilon_0} \left(r - \frac{a^2}{r} \right) \qquad a \leq r \leq b$$

(c) For $r \geq b$, the cylindrical Gaussian surface is outside the shell. The charge inside is obtained using only the volume that contains charge :

$$Q_{inside} = \rho V_{inside}$$
$$= \rho(\pi b^2 L - \pi a^2 L)$$

So Gauss' Law gives us

$$E \times 2\pi r L = \frac{\rho \pi L}{\epsilon_0}(b^2 - a^2)$$

and

$$E = \frac{\rho \pi L}{2\pi r L \times \epsilon_0}(b^2 - a^2)$$

$$= \frac{\rho}{2\epsilon_0 r}(b^2 - a^2)$$