Name of College - S. S. College, J-Bad
Dept - Mathematics
Topic - Transpose and Tranjugate of a Matrix (Contd)
Class - B-Sc I (Sub + HONS)
Time — 1:00 pm to 1:45 pm
Date - 15-09-2020

By- Samarendra Kumar

Hermitian Matrix

A Square Matrix $A = [a_{ij}]$ is called Hermitian if $A = A^*$

i.e if the Tranjugate of a Matrix A is itself A, then A is said to be Hermitian.

or A is said to be Hermitian if

$$a_{ij} = \overline{a_{ji}} \quad \forall i \text{ and } j$$

Ex:-> $A = \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix} \Rightarrow \overline{A} = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$

$$\overline{A}^T = \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix} = A$$

$\therefore A^* = A \Rightarrow A$ is Hermitian Matrix

Show that- the Matrix

$\begin{bmatrix} 1 & 1-i4 \\ 1+i4 & 2 \end{bmatrix}$ is Hermitian Matrix (2)

Here $A = \begin{bmatrix} 1 & 1-i4 \\ 1+i4 & 2 \end{bmatrix}$

$\Rightarrow A^T = \begin{bmatrix} 1 & 1+i4 \\ 1-i4 & 2 \end{bmatrix}$

$(\overline{A^T}) = \begin{bmatrix} 1 & 1-i4 \\ 1+i4 & 2 \end{bmatrix} = A$

$\Rightarrow \underline{A^\theta = A}$ Thus A is Hermitian Matrix

Q Show that $\begin{bmatrix} 3 & 7-4i & -2+5i \\ 7+4i & -2 & 3+i \\ -2-5i & 3-i & 4 \end{bmatrix}$

Here $A = \begin{bmatrix} 3 & 7-4i & -2+5i \\ 7+4i & -2 & 3+i \\ -2-5i & 3-i & 4 \end{bmatrix}$

$\Rightarrow A^T = \begin{bmatrix} 3 & 7+4i & -2-5i \\ 7-4i & -2 & 3-i \\ -2+5i & 3+i & 4 \end{bmatrix}$

$\Rightarrow \overline{A^T} = \begin{bmatrix} 3 & 7-4i & -2+5i \\ 7+4i & -2 & 3+i \\ -2-5i & 3-i & 4 \end{bmatrix} = A$

$\therefore A^\theta = A \Rightarrow A$ is Hermitian Matrix

Show that A is Hermitian Matrix

where $A = \begin{bmatrix} 1 & 1+i & 2+3i \\ 1-i & 2 & -i \\ 2-3i & i & 0 \end{bmatrix}$

$$A^T = \begin{bmatrix} 1 & 1-i & 2-3i \\ 1+i & 2 & i \\ 2+3i & -i & 0 \end{bmatrix}$$

$$\left[\overline{A^T}\right] = \begin{bmatrix} 1 & 1+i & 2+3i \\ 1-i & 2 & -i \\ 2-3i & i & 0 \end{bmatrix} = A$$

Thus $A^\theta = A \Rightarrow A$ is Hermitian Matrix

Remarks $\rightarrow$ Symmetric Matrix is a particular case of Hermitian Matrix if all the elements are real.

2. All positive integral power of a Hermitian Matrix is Hermitian Matrix.

Skew Hermitian Matrix :$\rightarrow$ A Square Matrix A is said to be Skew Hermitian if $A = -A^\theta$

A is said to be Skew-Hermitian if

$a_{ij} = -\bar{a}_{ji} \; \forall \; i$ and j

Show that

$\begin{bmatrix} 3i & 2+i \\ -2+i & i \end{bmatrix}$ is Skew Hermitian

Here $A = \begin{bmatrix} 3i & 2+i \\ -2+i & i \end{bmatrix}$

$\Rightarrow A^T = \begin{bmatrix} 3i & -2+i \\ 2+i & i \end{bmatrix}$

$\Rightarrow \overline{(A^T)} = \begin{bmatrix} -3i & 2-i \\ -2-i & -i \end{bmatrix}$

$= -\begin{bmatrix} 3i & -2+i \\ 2+i & i \end{bmatrix}$

$A^* = -A$

$\Rightarrow A = -A^*$ $\Rightarrow$ the given Matrix is Skew Hermitian Matrix

Show that the Matrix A is Skew Hermitian where

$A = \begin{bmatrix} 0 & 1+3i & 5 \\ -1+3i & 0 & -1+2i \\ -5 & 1+2i & -3i \end{bmatrix}$

Here

$$A = \begin{bmatrix} 0 & 1+3i & 5 \\ -1+3i & 0 & -1+2i \\ -5 & 1+2i & -3i \end{bmatrix}$$

$$A^T = \begin{bmatrix} 0 & -1+3i & -5 \\ 1+3i & 0 & 1+2i \\ 5 & -1+2i & -3i \end{bmatrix}$$

$$\overline{A^T} = \begin{bmatrix} 0 & -1-3i & -5 \\ 1-3i & 0 & 1-2i \\ 5 & -1-2i & 3i \end{bmatrix}$$

$$= - \begin{bmatrix} 0 & 1+3i & 5 \\ -1+3i & 0 & -1+2i \\ -5 & 1+2i & -3i \end{bmatrix}$$

i.e. $A^* = -A$

$\Rightarrow A = -A^*$

Hence Given Matrix is Skew Symmetric Hermitian Matrix