Name of College - S-S-College J-Bad

Deptt - Mathematics

TOPIC - Derivatives of Arc-length in Cartesian Co-ordinate

Class - B-Sc I (Hons)

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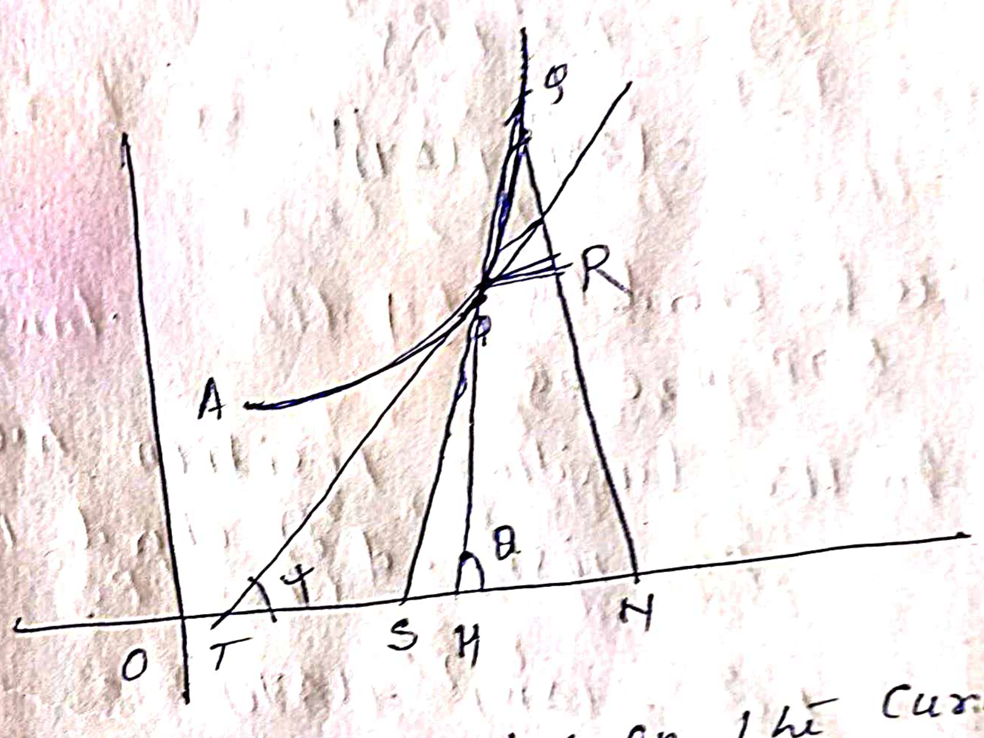
Study Material

Derivatives of Arc-length in Cartesian Co-ordinate System ⟶



Let $P(x,y)$ be any point on the curve.

Let $Q(x+\Delta x, y+\Delta y)$ be any other point which is very close to $P(x,y)$ on the curve

From P Draw PM and from Q Draw QN perpendicular to x-axis respectively

then $OM = x$ $\quad ON = x+\Delta x$

$PM = y$ $\quad NQ = y+\Delta y$

PR is perpendicular to QN Drawn From P

$\therefore$ $MN = ON - OM = x + \Delta x - x = \Delta x = PR$

$QR = QN - RN = y + \Delta y - y = \Delta y$

Let A be any point on the curve

such that $\overset{\frown}{AP} = s$

$\overset{\frown}{AQ} = s + \Delta s$

$\therefore$ $\overset{\frown}{PQ} = \Delta s$

Now From ΔPQR

$$PQ^2 = PR^2 + RQ^2$$

$$= (\Delta x)^2 + (\Delta y)^2$$

Since $\lim_{Q \to P} \dfrac{\text{Chord } PQ}{\text{Arc } PQ} = 1$

In the Limiting position we can write

$\overset{\frown}{PQ}$ instead of chord PQ.

$$\Rightarrow \Delta s^2 = (\Delta x)^2 + (\Delta y)^2$$

$$\left(\frac{\Delta s}{\Delta x}\right)^2 = 1 + \left(\frac{\Delta y}{\Delta x}\right)^2$$

Taking the Limit

$$\left(\frac{ds}{dx}\right)^2 = 1 + \left(\frac{dy}{dx}\right)^2$$

$$\therefore \frac{ds}{dx} = \sqrt{1 + y_1^2}$$

Again

$$ds \quad \left(\frac{\Delta s}{\Delta y}\right)^2 = \left(\frac{\Delta x}{\Delta y}\right)^2 + 1$$

Taking Limit -

$$\left(\frac{ds}{dy}\right)^2 = \cancel{1+\left(\frac{dx}{dy}\right)^2} \; 1 + \left(\frac{dx}{dy}\right)^2$$

$$\therefore \frac{ds}{dy} = \sqrt{1 + \left(\frac{dx}{dy}\right)^2}$$

Again $$\left(\frac{ds}{dt}\right)^2 = \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2$$

* Angle between the Tangent and x - axis

Here say the Equation of Given Curve be $y = f(x)$

Let $P(x, y)$ be any point on the Curve

PT has be drawn Tangent from P which meets OX in T. Let the angle between PT and x - axis $= \psi$

Produce QP which meets the x - axis

at the point S.

Let $\angle PSX = \theta$ $\therefore$ $\angle QPR = \theta$.

Now $\sin QPR = \frac{QR}{PQ}$

$\Rightarrow \sin\theta = \frac{\Delta y}{PQ}$ —— (1)

$\cos\theta = \frac{\Delta x}{PQ}$

$\tan\theta = \frac{\Delta y}{\Delta x}$

Now let us suppose $Q \to P$.

then in this case,

the chord $PQ \to$ tangent at P.

Hence $\theta \to \psi$

Also $\Delta x \to 0$

and $\lim_{Q \to P} \frac{\text{Chord } PQ}{\text{Arc } PQ} \to 1$

$\therefore$ Chord $PQ \to \Delta S$

Hence in the limiting position

when $Q \to P$

we have $\sin\psi = \frac{dy}{ds}$

$\cos\psi = \frac{dx}{ds}$

$\tan\psi = \frac{dy}{dx}$

Remarks —

If tangent $\parallel$ x-axis

$\frac{dy}{dx} = 0$

If tangent $\parallel$ y-axis

$\frac{dx}{dy} = 0$